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EFFECT OF JOINT DIMENSIONS ON STRENGTH AND STIFFNESS OF TENON JOINTS

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SYNOPSIS. The influence of joint dimensions on the bending strength and stiffness of tenon joints was investigated. Factors such as the tenon thickness, length and width were considered. The regression functions for the strength and stiffness of the joint were calculated.

KEY WORDS: tenon joint, tenon dimensions, bending strength, stiffness, regression formula

INTRODUCTION

Tenon joints are still widely used in wooden constructions. Despite the increasing use of dowel joints, they are irreplaceable for some types of furniture constructions.

A number of investigations of the tenon joint strength has been made. Effects of such factors as: the tolerance between the tenon and the mortise, the kind of adhesive used to assembly, the way of adhesive coating, the species of wood and the moisture content of wood were investigated. Important factors are also the tenon dimensions. Several studies have been only made on the effect of these dimensions. In the research carried out in FIRA (Sparkes 1968) the influence of tenon width and length was evaluated, however, it was limited to two or three values of these dimensions only. More comprehensive investigations were carried out by Hill and Eckelman (1973) who determined not only the strength but also the stiffness of tenon joints. They evaluated the effect of tenon length, which varied from 0.5 to 2.0 inches, with the constant tenon width of 2.0 inches. In turn, evaluating the effect of tenon width, which varied from 0.5 to 3.0 inches, they kept the constant tenon length of 1.0 inch. The influence of tenon width and length was also investigated by Kamenicky (1975), but he limited himself to determine the joint flexibility only.

The above described studies were characterized by the fact that the influences of tenon length and width were investigated separately. The Institute of Technology of Bydgoszcz University has carried out research on joinery joint in which a simultaneous effect of several joint dimensions is regarded. The previous study by the authors (Warmbier and Wilczyński 2000) was concerned with dowel corner joints. The objective of this study was to determine the simultaneous effect of tenon dimensions on the strength and stiffness of tenon joints.

It is worth mentioning that the mechanical problems of tenon joints are still not satisfactorily resolved despite the investigations which have been carried out for many years. For example Smardzewski in his last paper (Smardzewski 2002) analyzed stresses in different places of this joint. The knowledge of these stresses should be useful for the rational joint design, likewise the knowledge about the influence of joint dimensions on the strength and stiffness of the joints.

MATERIALS AND METHODS

The general configuration of the tenon joint used in this study is illustrated in Figure 1. It is the joint of the corner type. Both members of the joint specimen were of the same thickness and width. Beech wood (*Fagus silvatica* L.) with the average density of 720 kg/m^3 and the moisture content of 8% was used as the material of the joint members.

The mortises were machined on a drilling and milling machine using drills whose diameters were 0.2 mm larger than the tenon nominal thickness. The depth of the mortise was 2 mm deeper than the tenon length. The tenons were machined on an underdrive spindle shaper. Tenon thickness was measured and pairs of joint members were matched to get the mortise-tenon clearance of 0.2 mm. The specimens were assembled with a polyvinyl acetate adhesive. Both the mortise and tenon were coated liberally with glue. A piece of wax paper was slipped over the end of the tenon before assembly to prevent the adhesive from bonding the joint members.

The tenon thickness t, length l and width w (Fig. 1) were assumed as variable factors. Four or five values of each dimension were taken into account so that non-linear regression functions for the strength and stiffness of the joint could be determined. The tenon thicknesses were: 6, 8, 10 and 12 mm; the tenon lengths: 10, 18, 26, 34 and 42 mm; the tenon widths: 18, 26, 34, 42 and 50 mm. Because of a large number of possible combinations, not a complete but a selected plan of tests was adopted (Polański 1984). It consisted of 32 selected test combinations.

Prior to the tests, test specimens were allowed to cure for two weeks in a controlled climate room which maintained an equilibrium moisture content of 7.5 to 8.0% in the wood. Thirty two sets of specimens consisting of ten specimens each were prepared altogether.

Corner joints can be loaded in several ways (Wilczyński and Warmbier 1996). The tension loading as shown in Figure 2 was used. The forces P tend to open the joint and cause the bending moment to appear in the corner of the joint:

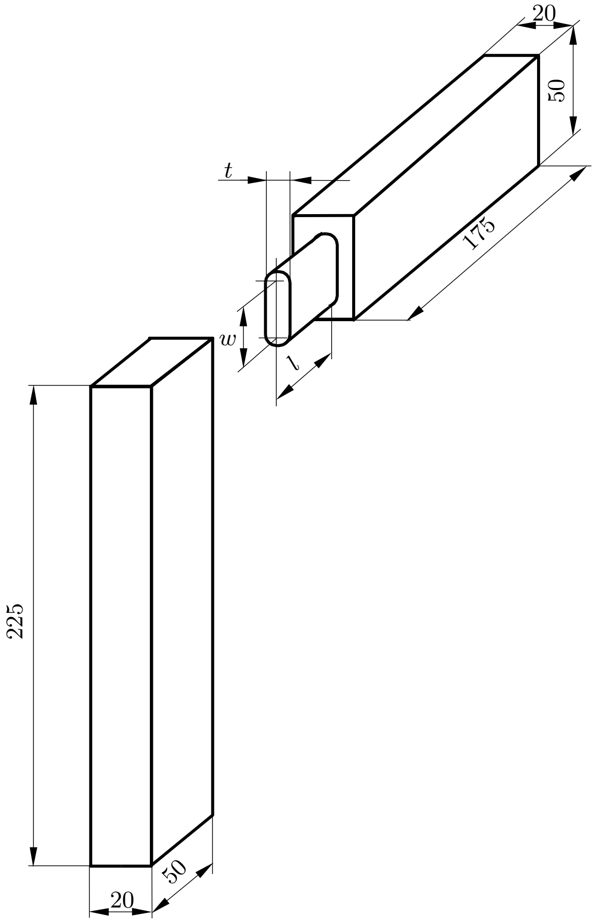

Fig. 1. Geometry and dimensions of tenon joint specimens: t – tenon thickness, l – tenon length, w – tenon width
Rys. 1. Kształt i wymiary próbek połączenia o złączu czopowym: t – grubość czopa, l – długość czopa, w – szerokość czopa

$$M = 0.707Pa \tag{1}$$

where: M – bending moment [Nm],
P – applied force [N],
a – distance from the point of force application to the middle of the corner of the joint [m].

The angle of a relative rotation of joint members was assumed as a characteristic deformation of the studied joint. For small elastic deformations the relationship of the moment to the angle of rotation is linear:

$$M = k\gamma \tag{2}$$

where: k – coefficient of joint rigidity [Nm/rad],
γ – angle of relative rotation of joint members [rad].

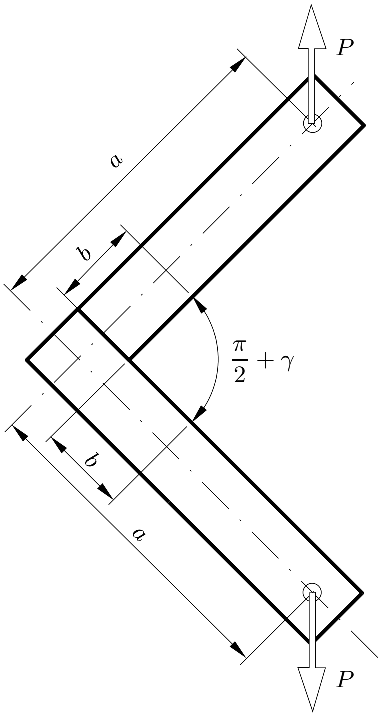

Fig. 2. Diagram of loading of test specimen and measuring of angle γ of relative rotation of specimen members: $a = 175$ mm, $b = 30$ mm

Rys. 2. Schemat obciążania i pomiaru kąta γ względnego obrotu ramion próbki: $a = 175$ mm, $b = 30$ mm

The angle γ was measured on the arm b of 30 mm long (Fig. 2). The assumption of a relatively short arm of the angle was aimed at minimizing the effect of the flexion of the joint members. The joint rigidity coefficient k was taken as a characteristic of the joint stiffness. It was calculated by means of the following formula:

$$k = \frac{\Delta M}{\Delta \gamma} \tag{3}$$

where: ΔM – moment increment [Nm],
$\Delta \gamma$ – rotation angle increment caused by the moment increment [rad].

The moment increment was defined as:

$$\Delta M = M_2 - M_1 \tag{4}$$

where: M_1 – was about 10% and M_2 was about 40% of the mean value of the bending strength for a given test combination.

The bending strength M_f was calculated from the formula (1) by measuring the force P in which the joint failed. All tests were carried out on a universal testing machine at a rate of loading of 2 mm per minute.

RESULTS

The results of the tests – values of the bending strength and the joint rigidity coefficient for all joint specimens were subjected to a regression analysis. The computer program CADEX – ESDET (Polański and Górecka-Polańska 1992), which allows to calculate regression functions of different mathematical form using the method of least squares, was employed. After the analysis of the functions a second power polynomial with interactions turned out to be the most fitted to the results of the tests both for the strength and the stiffness of the joint. The following formulas were obtained:

$$\begin{aligned} M_f &= 119.44 - 9.767\,t + 5.756\,l - 6.725\,w + 0.5356\,t^2 - \\ &- 0.1581\,l^2 + 0.1150\,w^2 + 0.2888\,tl - 0.0971\,tw + 0.1678\,lw \end{aligned} \qquad (5)$$

$$\begin{aligned} k &= 3970 - 424.8\,t + 45.78\,l - 232.7\,w + 8.221\,t^2 - \\ &- 2.814\,l^2 + 3.969\,w^2 + 5.705\,tl + 15.61\,tw + 4.396\,lw \end{aligned} \qquad (6)$$

where: M_f – bending strength [Nm],
k – rigidity coefficient [Nm/rad],
t – tenon thickness [mm],
l – tenon length [mm],
w – tenon width [mm].

Based on the formulas (5) and (6), diagrams were drawn to present the relationship of the bending strength (Fig. 3-5) and the rigidity coefficient (Fig. 6-8) of the joint to the joint dimensions: the thickness, length and width of the tenon. The joint strength increases as the tenon dimensions increase. The effect of tenon thickness is shown in Figure 3. The increase in the strength due to the increase in the tenon thickness is the greater, the greater tenon length is and as the smaller tenon width is. The increase in the tenon thickness from 6 to 12 mm results in the increase in the joint strength on the average by 10.1%. It can be said that the dependence of the joint strength on the tenon thickness is slight.

The effect of the tenon length on the joint strength is shown in Figure 4. This strength increases less and less with increasing the tenon length, especially for smaller tenon widths and smaller tenon thickness. For example, the strength of the joint with the 8 mm tenon thickness and the 34 mm tenon width increases by 75.2, 54.6, 34.0 and 13.5 Nm due to increase in the tenon length from 10 to 18 mm, from 18 to 26 mm, from 26 to 34 mm and from 34 to 42 mm, respectively. The bigger the tenon width and the tenon thickness is, the bigger the influence of the tenon length on the joint strength is. The increase in the tenon length from 10 to 42 mm results in the increase in the joint strength on the average by 227.6%. It can be concluded that the joint strength depends very strongly on the tenon length.

The dependence of the joint strength on the tenon width differs from that on the tenon length. The joint strength increases more and more with increasing the tenon width, especially for bigger tenon lengths (Fig. 5). The bigger the tenon length is, the bigger the effect of the tenon width is. The average increase in the joint strength caused by increasing the tenon width from 18 to 50 mm is 98.0%. Therefore, the effect of the tenon width is significant but less than that of the tenon length.

The stiffness of the tested joints, like their strength, increases as the tenon dimensions increase. The influence of the tenon thickness on the coefficient of joint rigidity is approximately linear (Fig. 6). The increase in this coefficient due to the increase in the tenon thickness is the bigger as the bigger tenon width is and is almost independent on the tenon length. The increase in tenon thickness from 6 to 12 mm results in the increase in the joint rigidity coefficient on the average by 39.5%.

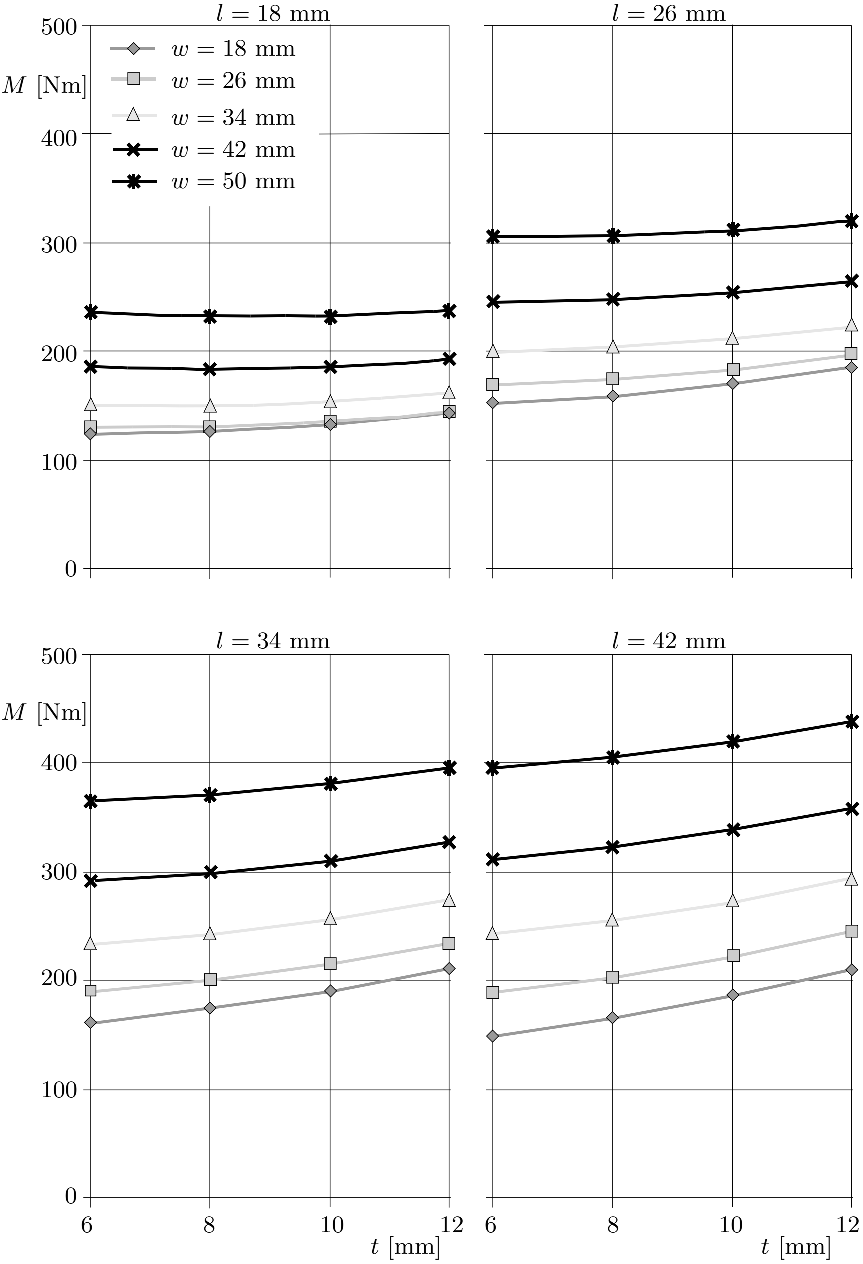

Fig. 3. Relationship of joint bending strength (M_f) to tenon thickness (t) for different tenon widths (w) and lengths (l)
Rys. 3. Zależność wytrzymałości na zginanie połączenia (M_f) od grubości czopa (t) dla różnych szerokości (w) i długości czopa (l)

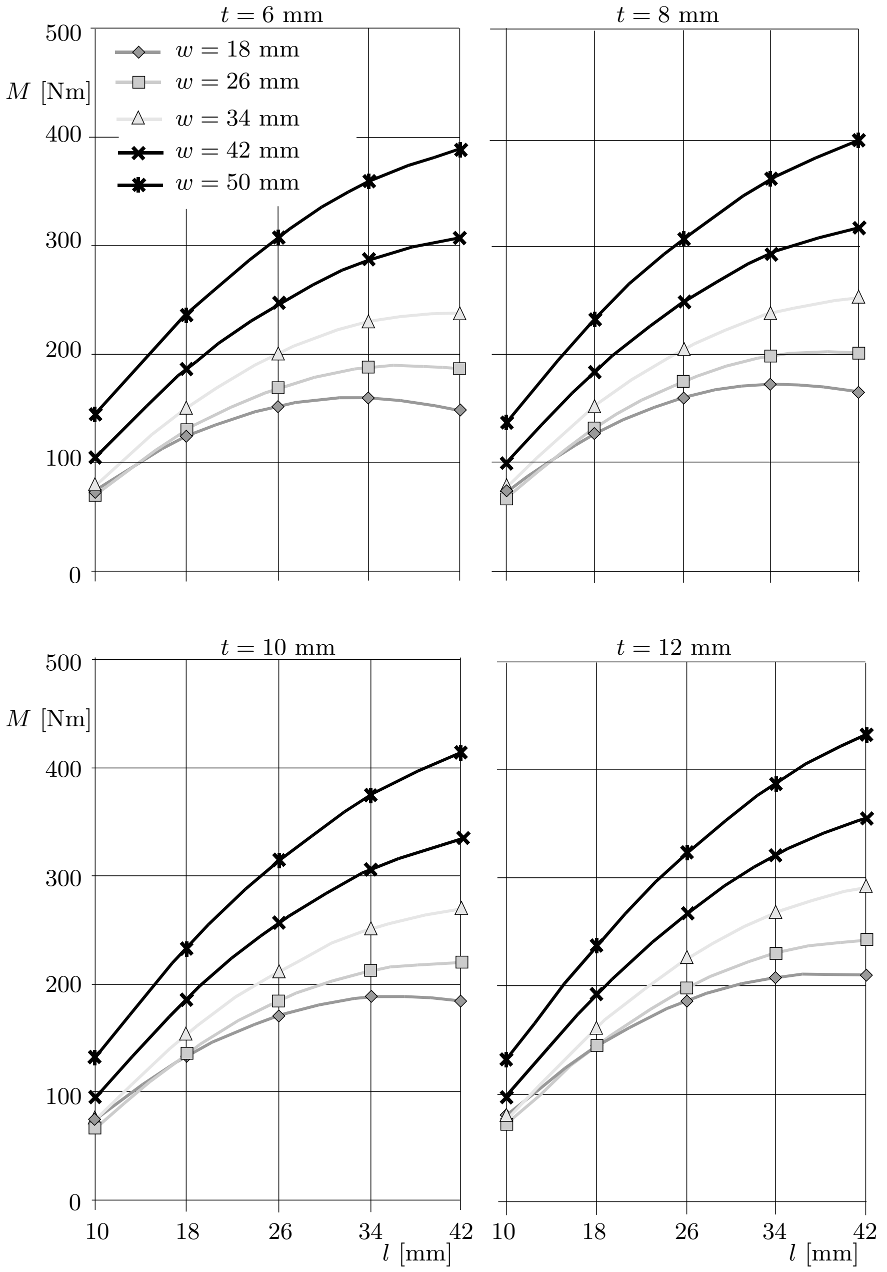

Fig. 4. Relationship of joint bending strength (M_f) to tenon length (l) for different tenon widths (w) and thicknesses (t)

Rys. 4. Zależność wytrzymałości na zginanie połączenia (M_f) od długości czopa (l) dla różnych szerokości (w) i grubości czopa (t)

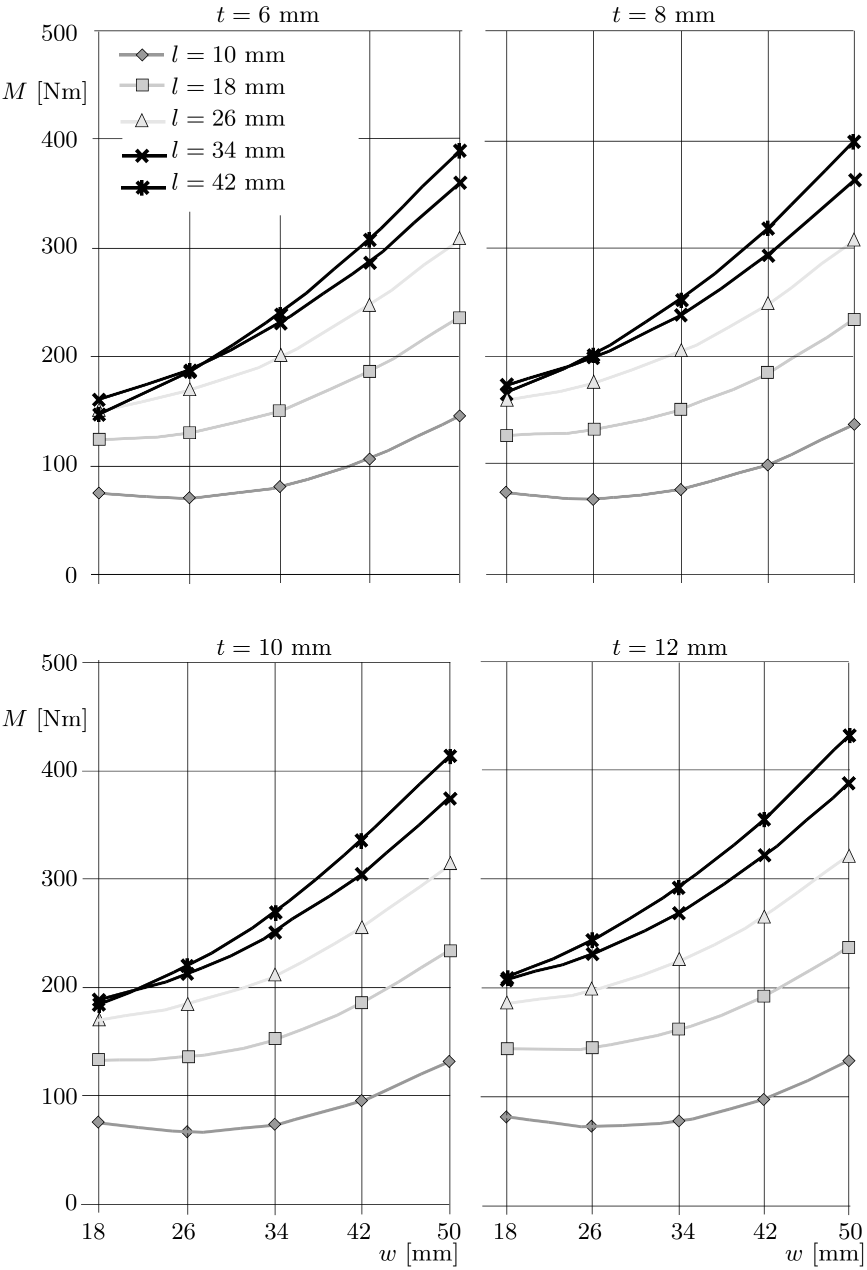

Fig. 5. Relationship of joint bending strength (M_f) to tenon width (w) for different tenon lengths (l) and thicknesses (t)
Rys. 5. Zależność wytrzymałości na zginanie połączenia (M_f) od szerokości czopa (w) dla różnych długości (l) i grubości czopa (t)

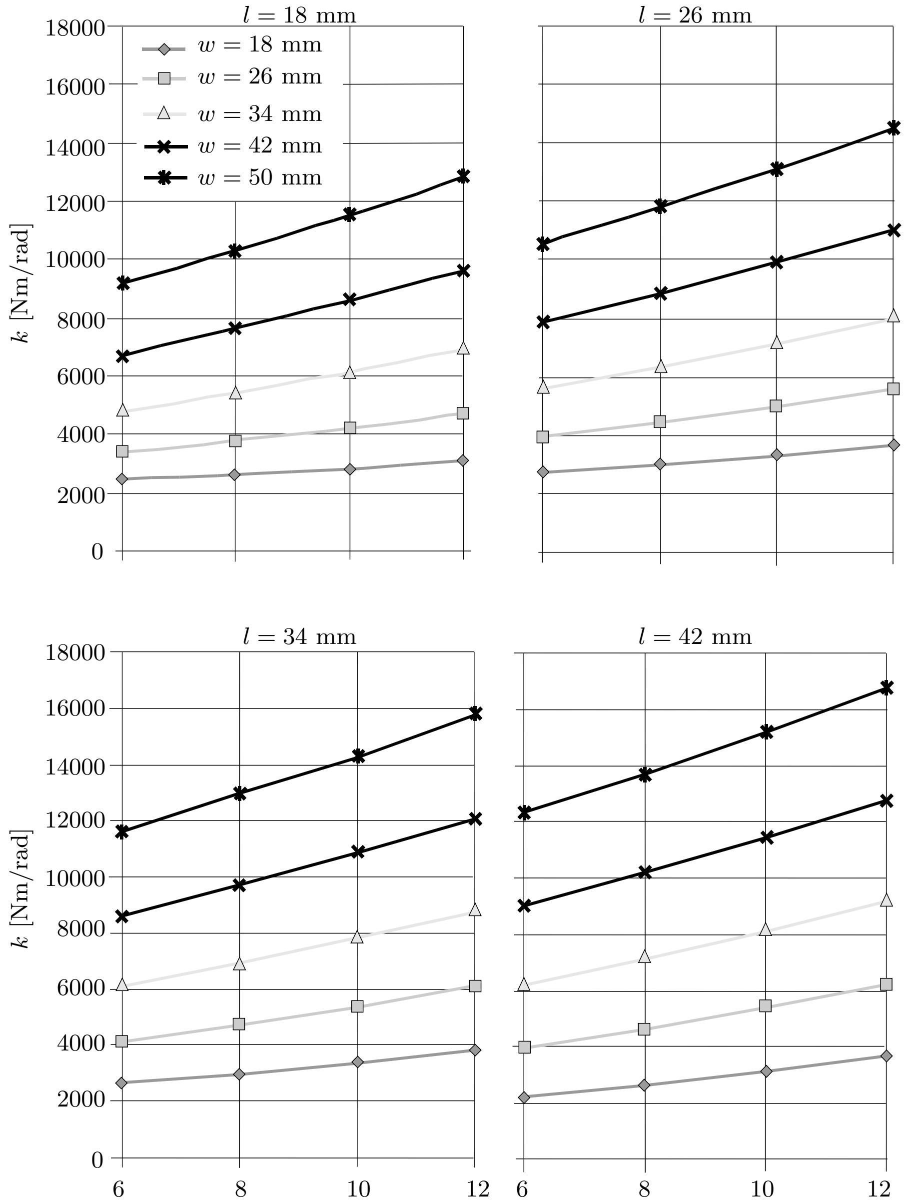

Fig. 6. Relationship of joint rigidity coefficient (k) to tenon thickness (t) for different tenon widths (w) and lengths (l)

Rys. 6. Zależność współczynnika sztywności połączenia (k) od grubości czopa (t) dla różnych szerokości (w) i długości czopa (l)

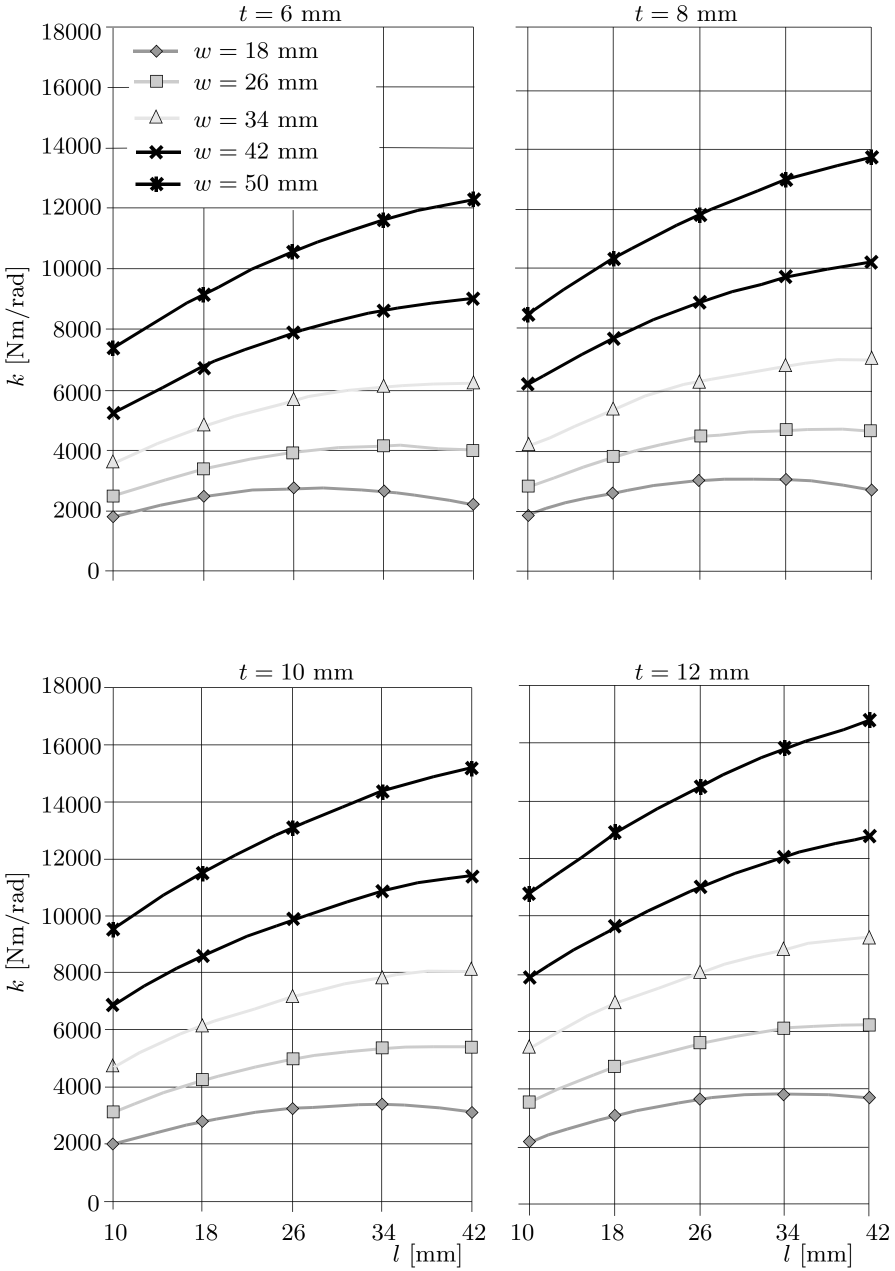

Fig. 7. Relationship of joint rigidity coefficient (k) to tenon length (l) for different tenon widths (w) and thicknesses (t)

Rys. 7. Zależność współczynnika sztywności połączenia (k) od długości czopa (l) dla różnych szerokości (w) i grubości czopa (t)

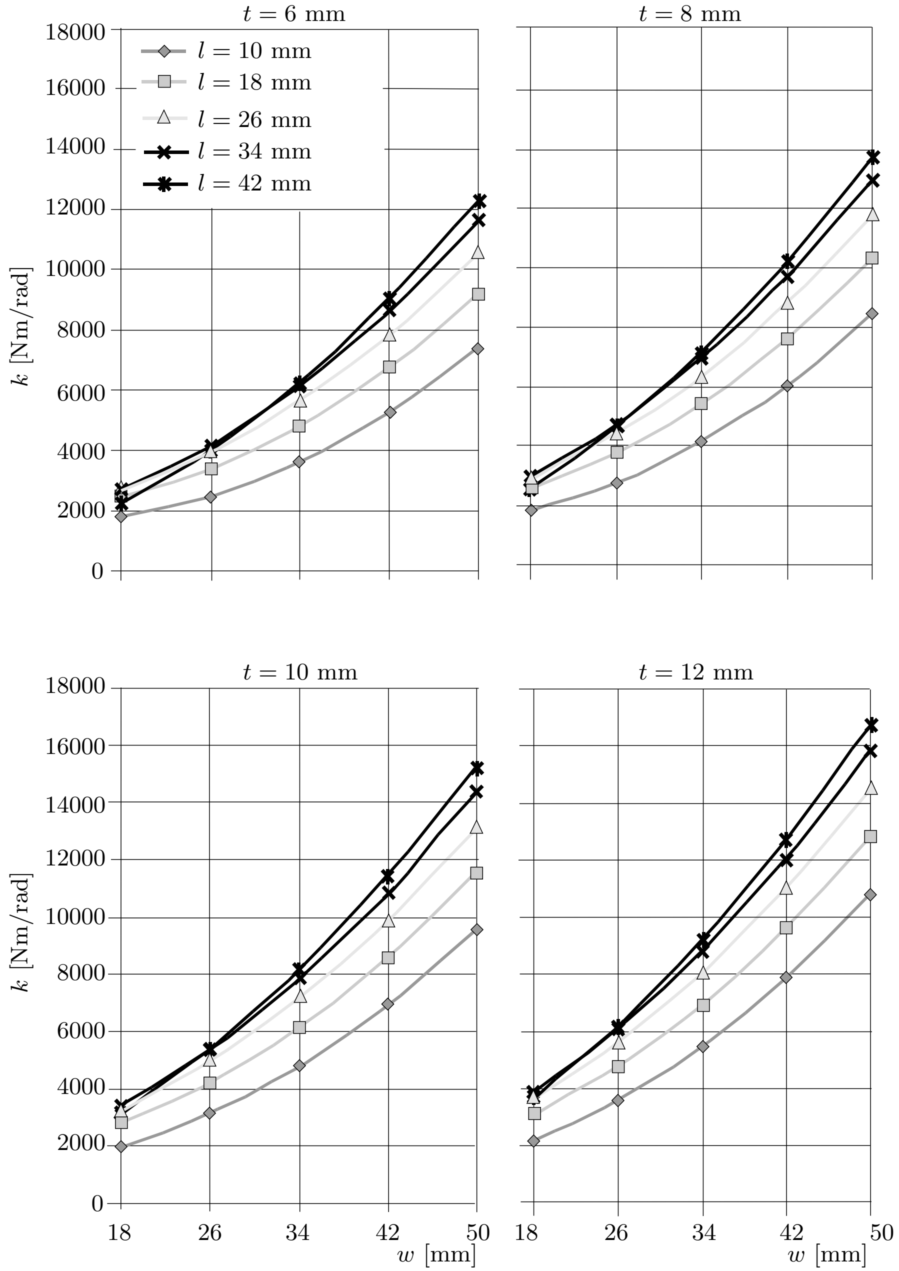

Fig. 8. Relationship of joint rigidity coefficient (k) to tenon width (w) for different tenon lengths (l) and thicknesses (t)

Rys. 8. Zależność współczynnika sztywności połączenia (k) od szerokości czopa (w) dla różnych długości (l) i grubości czopa (t)

Joint rigidity coefficient – tenon length diagrams are shown in Figure 7. A gradual increase in the tenon length results in the smaller and smaller increase in the rigidity coefficient, especially for smaller tenon widths. In the case of the tenons with the 18 mm width this coefficient even decreases as the tenon length is over 34 mm. The greater the tenon width is, the greater the influence of the tenon length on the joint rigidity coefficient is. The increase in the tenon length from 10 to 42 mm results in the increase in this coefficient on the average by 62.3%.

The dependence of the joint stiffness on the tenon width is shown in Figure 8. The joint rigidity coefficient increases more and more with increasing the tenon width. The increase in this coefficient due to the increase in the tenon width is the bigger, the bigger tenon length and thickness is. The average increase in the rigidity coefficient caused by increasing the tenon width from 18 to 50 mm is 336.5%. Therefore, the influence of the tenon width on the joint stiffness is much bigger than that of the tenon length and thickness.

Summarizing the results analysis one has to note that tenon length and width have very big influence on the strength and stiffness of the tenon joint. The length has a more significant effect on the joint strength, whereas the width on the joint stiffness.

The formulas (5) and (6) are very complex and inconvenient for practical use. Therefore, the following equations in a form of power functions product were calculated:

$$M_f = 1.328\, t^{0.167}\, l^{0.743}\, w^{0.648} \tag{7}$$

$$k = 4.614\, t^{0.512}\, l^{0.326}\, w^{1.453} \tag{8}$$

were: M_f [Nm], k [Nm/rad], t, l and w [mm].

The calculated regression functions for the strength and stiffness of the tenon joint present very wide possibilities for a wood structures designer – from determining mechanical properties of a joint with given joint dimensions to choosing alternative combinations of a joint dimensions that ensure the required strength and stiffness of the joint. However, it is important to remember that the obtained formulas are valid for the given materials only: beech members glued with the use of the polyvinyl acetate adhesive with the necessary mortise – tenon clearance.

CONCLUSIONS

1. The influence of joint dimensions on mechanical properties of the tenon joint can be expressed by means of regression functions in the form of a second power polynomial with interactions or in the form of a power functions product.
2. The bending strength and stiffness of the joint increase as the tenon dimensions increase.
3. The greatest effect on the joint strength has the tenon length, the influence of the tenon width is less significant and the effect of the tenon thickness is slight.

4. The joint stiffness depends first of all on the tenon width, the effects of the tenon length and thickness are less significant.

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WPŁYW WYMIARÓW ZŁĄCZA NA WYTRZYMAŁOŚĆ I SZTYWNOŚĆ POŁĄCZENIA O ZŁĄCZU CZOPOWYM

Streszczenie

W pracy przedstawiono badania dotyczące wpływu wymiarów złącza na wytrzymałość i sztywność połączenia kątowego płaskiego o złączu czopowym. Próbki połączeń wykonano z drewna bukowego, używając do ich montażu kleju polioctanowinylowego. Czynnikami zmiennymi były: grubość, długość i szerokość czopa. Posługując się selekcyjnym planem badań, określono eksperymentalnie wytrzymałość i sztywność połączenia dla wybranych kombinacji tych czynników. Wyprowadzono funkcje regresji dla wytrzymałości i sztywności połączenia. Mają one postać wielomianu drugiego stopnia z interakcjami lub iloczynu funkcji potęgowych. Największy wpływ na wytrzymałość połączenia wywiera długość czopa, nieco mniejszy szerokość czopa, a znikomy – grubość czopa. Sztywność połączenia zależy przede wszystkim od szerokości czopa, w mniejszym stopniu od jego długości i grubości.

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