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CONSTRUCTION OPTIMISATION OF UPHOLSTRED FURNITURE

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The aim of this research project was optimisation of the supporting structure of a single-seat armchair with respect to the stability and cross section dimensions of all of its elements. The optimisation was carried out with the use of numeric method using Algor computer program. True mechanical properties of materials used and levels of normal stress caused were taken into account in the method. The new model of the supporting structure retained its stability and guaranteed optimal strength parameters.

Key words: computer, optimization, strength, stability

INTRODUCTION

According to European standards, modern furniture has to comply with rigorous quality requirements concerning its function, aesthetics and strength. On the other hand, furniture production should bring maximum profit and requires minimum production cost. However, these costs are directly influenced by the amount of raw materials, machines and production resources used in the production cycle. From the structural point of view, the problem of rational investment of financial resources in the course of the designing process may be solved by optimisation of material utilization. Material optimisation of the construction should direct the designer's work towards cost reduction, but, at the same time, assure adequate rigidity and strength of the furniture. The optimisation process requires good knowledge of problems associated with the choice of shape parameters and physical and mechanical properties of construction

elements. Therefore, the aim of furniture construction optimisation is to (Brdyś and Ruszczyński 1985, Dzięgielewski and Smardzewski 1995, Ostwald 1987):
- minimize production costs, that is material consumption, working time of machines and employees,
- maximize construction rigidity,
- maximize the strength of construction elements and joints,
- assure adequate stability of individual elements and the entire construction.

Optimisation criteria are basically expressed by the function of decision variables Q=f(x), for which objective function reaches extreme values.

$$Q=f(x_1,x_2,x_3,...x_n)\Rightarrow \text{minimum} \quad (1)$$

So far, the subject of optimisation of furniture construction was limited to the aspect of finding the best engineering solutions in order to minimize material consumption and maximize rigidity and strength of box-type or carcass furniture (Dzięgielewski and Smardzewski 1995, Smardzewski 1989, 1990, 1992). The problem of stability of back walls in box-type furniture (Smardzewski and Dzięgielewski 1992) or stability of bars in frame constructions (Gustafson 1996 a,b) was dealt with only in very few cases. Several successful attempts were also made in order to improve elasticity of upholstered elements of beds and armchairs by designing optimum solutions for the shape and dimensions of springs or foam-spring systems in upholstered furniture (Smardzewski and Grbac 1998). Optimisation of this type of furniture also required gradual solving of problems concerning construction rigidity and strength of flat upholstery frames (Kapica et al. 1983, Kapica, Korgól and Malinowski 1989, Kapica and Smardzewski 1992). So far, the problems of quality verification of supporting elements in armchairs, sofas and beds have not been undertaken. This concerns mainly constructions other than carcass furniture, namely, combinations of bar and board elements.

Because of the above-mentioned factors, this research project is an attempt to find an optimal solution for the construction of a supporting structure in a single-seat armchair.

OBJECT

The objective of this research project is to perform optimisation of the supporting structure in a single-seat armchair made of wood and chipboard produced in one of Polish furniture plants. The task consists in the calculation of armchair body stability and the selection of optimal sections for construction elements without decreasing their strength parameters. The task will be accomplished by means of a numeric analysis of the construction using a computer program performing calculations with a finite-element method. The project will result in a plan of construction adjustment implementation in order to reduce material consumption.

METHODS

The objective of analyses is the supporting structure in a single-seat armchair, of which the main module was constructed of wood and board elements joined with staples. Because of a large amount of staples used during the furniture assembly, it was assumed that all joints in the construction were stiff. Seat elements and bars used for bracing and joining the sides of the main module were made of birch wood. The cross section of these fillets was 25x50 mm. The sides of the main component and its front were prepared of 16 mm-thick chipboard. In addition, a wooden block of size 25x50x50 mm was fastened at each bar-board joint in order to increase the joint surface. The sides were also made of a 16 mm-thick chipboard and wrought birch fillets. In order to obtain esthetical, oval shape the sides were covered with upholstered cardboard fastened with staples. Each side was fastened to the main module using three screws. The construction diagram is shown in Fig. 1.

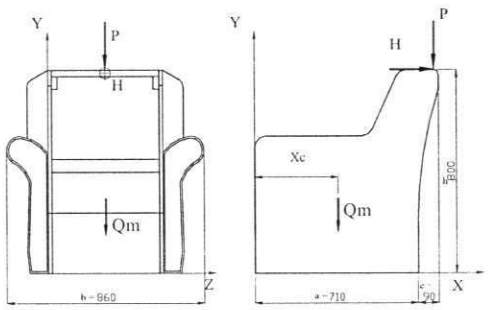


Fig.1. Diagram of armchair stability analysis in a symmetrical and asymmetrical system.
Rys.1. Schemat badania stateczności fotela w układzie symetrycznym i niesymetrycznym

The first stage of the construction improvement process was to determine its stability. Stability of a piece of furniture is a property which, when lost, may put at risk the health or life of its user in a sudden and direct manner. Determination of this property requires empirical tests or mathematical calculations. For comparison purposes, stability conditions in this research project were determined using both of these methods.

Laboratory tests were carried out by value assays of H horizontal force and P perpendicular force, for which balance of the armchair is disturbed. Then, after determining the shapes, dimensions and materials of separate elements, the coordinates of gravity centre were calculated and conditions for stability loss for the entire construction were determined. The diagram of the armchair stability analysis in an experimental way was shown in Fig. 1, for which X_c centre of gravity in an asymmetrical system was calculated from the following equation:

$$X_c = \frac{H \cdot h}{Q_m} \qquad (2)$$

where:

X_c – centre of gravity location
H – backrest height

Q_m – mass load total

H – perpendicular force value.

The analytical stability assessment consisted in the determination of the gravity centre for the construction and further calculation of critical forces causing balance disturbance of the armchair. Global centre of gravity for the armchair is a resultant of local gravity centres of individual construction elements (Fig. 2a). Therefore, after determining coordinates the gravity centre for the sides and the main component, the global centre of gravity for the whole supporting structure was calculated, according to the diagram shown in Fig. 2b.

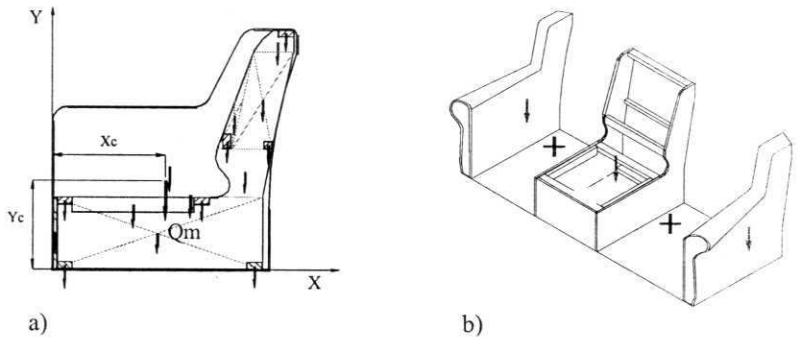


Fig.2. Diagram of gravity centre calculation for: a) components, b) whole construction
Rys.2. Schemat obliczania środka ciężkości dla: a) elementów składowych, b) całej konstrukcji

Coordinates of armchair gravity centre location were calculated on the basis of the following equations:

$$X_c = \frac{\sum X_i \cdot A_i}{\sum A_i} \tag{3}$$

$$Y_c = \frac{\sum Y_i \cdot A_i}{\sum A_i}$$

where:

X_i, Y_i – coordinates of local centres of gravity

A_i – mass of individual construction elements.

During everyday use, elements of an armchair incorporated in the supporting structure are mainly exposed to bending. Therefore, it was assumed that bending strength and modulus of elasticity at static bending are the best parameters to characterise bearing capacity of materials used. Thus, material optimisation of the construction was performed only for the main component (without sides), as this element is responsible for the basic strength functions. The optimised construction is shown in Fig. 3.

Strength optimisation was carried out by a numeric method using ALGOR computer program. The construction was prepared for tests in the following stages:

- Preparation of a numeric model using a three-dimensional grid with a scale of 5x5 mm.
- Supporting and weighting of the construction in selected joints

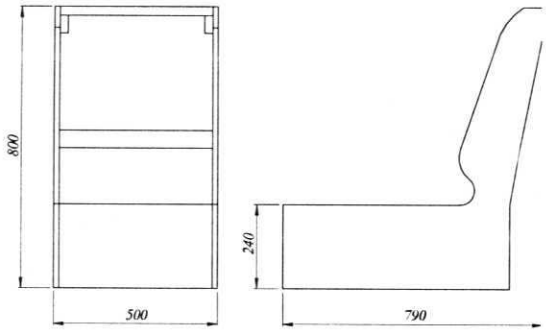


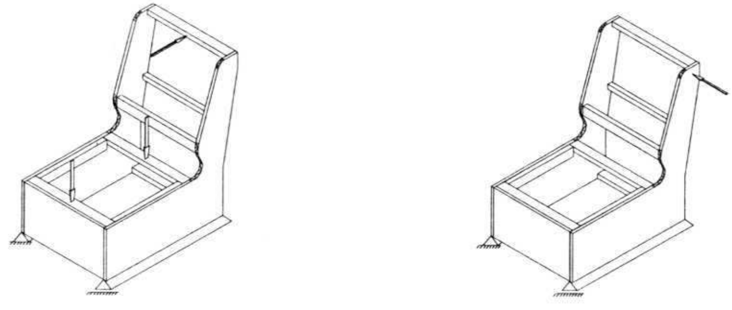
Fig.3. Main component of the armchair supporting structure
Rys.3. Część główna konstrukcji nośnej fotela

- Identification of mechanical properties of construction elements by the specification of density (ρ), modulus of density (E_g), Poisson's ratio (ν), modulus of rigidity (G)
- Construction decoding in the pre-processor
- Calculations in the processor
- Presentation of results in the graphic post-processor.

The main numerical processing was carried out for two static systems:

- Forces $P_1 = 650N$ and $P_2 = 700N$ applied to construction elements of the seat and backrest according to the diagram in Fig. 4a.
- Force $P_3 = 700N$ applied perpendicularly to the side in the upper part of the construction (Fig.4b) characterising an attempt to move the armchair on the ground.

The calculations were carried out using the obtained mean values of mechanical properties for materials utilised in the armchair production process (Table 1). Construction optimisation was carried out on the basis of the ordinary stress pattern calculated using the above method.



a) b)

Fig.4. Loading diagram for: a) seat and back rest, b) side of the armchair
Rys.4. Schemat obciążenia: a) siedziska i oparcia, b) boku fotela

Table 1

Tabela 1

Mechanical properties of construction materials

Właściwości mechaniczne materiałów konstrukcyjnych

Material Materiał	Density Gęstość ρ [kg/m3]	Bending strength Wytrzymałość na zginanie R_g [MPa]	Modulus of elasticity Moduł sprężystości E_g [MPa]	Poisson's ratio Współczynnik Poissona ν	Modulus of rigidity Moduł Kirhchoffa G [MPa]
Birch wood	650	102.8	14599.5	0.30	5600
Chipboard	700	13.2	2990.9	0.28	1172

RESULTS OF CALCULATIONS AND MEASUREMENTS OF THE ARMCHAIR CONSTRUCTION STABILITY

One of the aims of the performed optimisation was to find out whether the constructional design was correct with respect to spatial structure stability. According to methodological assumptions, this task was carried out through laboratory tests and analytic calculations. Values of critical forces causing loss of construction stability are equal: 13.2; 13.4; 13.7; 13.3; 13.4 [daN], together with the mean value of critical load.

The above values show that the loss of construction stability will take place after application of critical force H = 13.4 daN, where:

The value of perpendicular force P amounts to:

$$P = \frac{a}{e} \cdot Q_m = 233\ N \tag{4}$$

The value of the X_c coordinate of the centre of gravity in an asymmetrical system amounts to 362 mm.

Table 2 shows results of calculations concerning location of local gravity centres for individual construction elements and gravity centre coordinates for the complete armchair.

It should also be noticed that analytically calculated values of gravity centre location are similar to the values of gravity centre coordinates identified on the basis of experimentally measured critical force H. After identification of gravity centre location, calculations were carried out in order to determine critical force H, for which the balance of armchair is disturbed.

$$H = \frac{X_c \cdot Q_m}{h} = 13.76\ N \tag{5}$$

The value of the horizontal force H calculated using this method differs by less than 3% from that, evaluated in laboratory conditions assuring acceptable compliance of the experiment with theory. This allows to verify stability of furniture construction at the stage of the design office, because the precondition, which must be met to maintain stability, is the application of horizontal forces lower than the calculated critical forces.

Table 2

Tabela 2

Coordinates of gravity centre points

Współrzędne środka ciężkości

Element mass Masa elementu	Coordinates of local gravity centres Współrzędne lokalnych środków ciężkości	
A_i [kg]	X_i [mm]	Y_i [mm]
0.380	41	12.5
0.380	665	12.5
0.380	41	227.5
0.380	491	227.5
1.258	8	120
0.325	266	215
0.190	572.5	425
0.380	707.5	412.5
2.781	453	248
1.855	345	120
0.483	690	562
0.089	597	508
0.082	738	756
0.272	635	320
0.380	765	787.5
8.600	385	340
Whole construction Cała konstrukcja 29.600	372	296

STRESSES IN CONSTRUCTION ELEMENTS

Numerical calculations show that the highest stresses occurred in places where the load was applied and in narrowings of the construction sides. If a load of three concentrated forces is applied to a system, the accompanying stresses, are typical for the bending of beams supported at two points. The section, which was hazardous for these elements, occurred at half of the rail length and the highest stress was observed in the terminal fibres of the element (Fig. 5 and 6). The compression stress reached maximum values at the side, where the force was applied and highest values of tensile stress were observed symmetrically at the other side. Meanwhile, extremely high stress (values close to the board bending strength) occurred at the narrowing of the rack side (Fig. 7).

In the system of static loading of the construction with a force applied to the side of the rack, the highest values of stress were noted in the uppermost part of the board and in stud elements located above the level of the seat. Force $P_3 = 700N$ situated perpendicularly to the side caused stresses which were many times higher than forces P_1 and P_2 applied to the seat and back rest. The highest stresses exceeding bending strength of the chipboard occurred in places where the medium rails and uppermost rail were fastened to the side. In stud elements, the highest stresses also occurred in the layers, which were the closest to the joints of these elements. However, they were much below

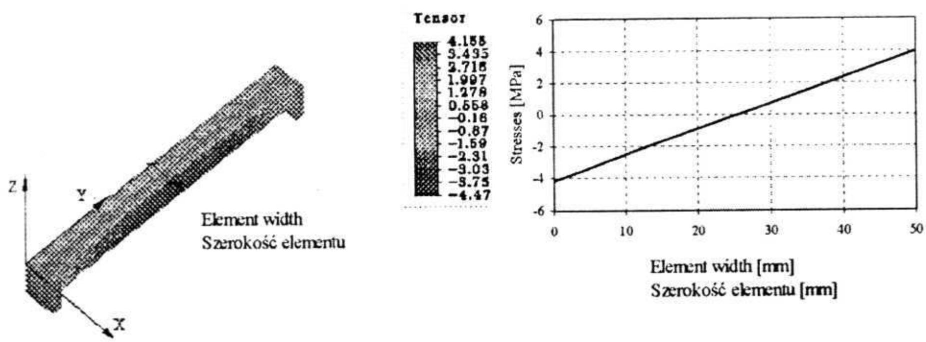


Fig. 5. Diagram of ordinary stresses in the uppermost rail
Rys.5. Obraz naprężeń normalnych w ramiaku górnym

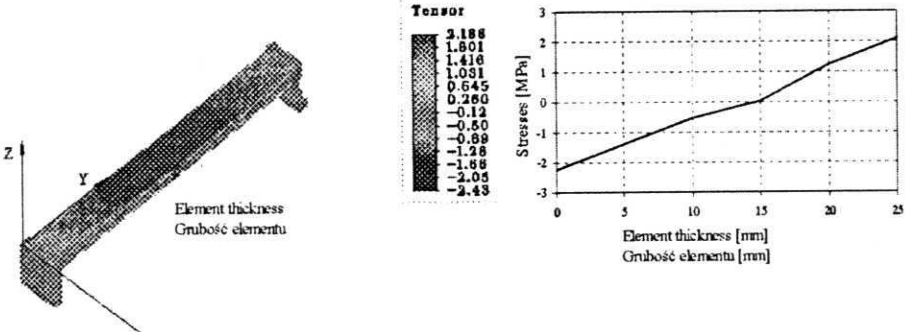


Fig. 6. Diagram of ordinary stresses in the posterior bearing of the seat
Rys.6. Obraz naprężeń normalnych w tylnej podporze siedziska

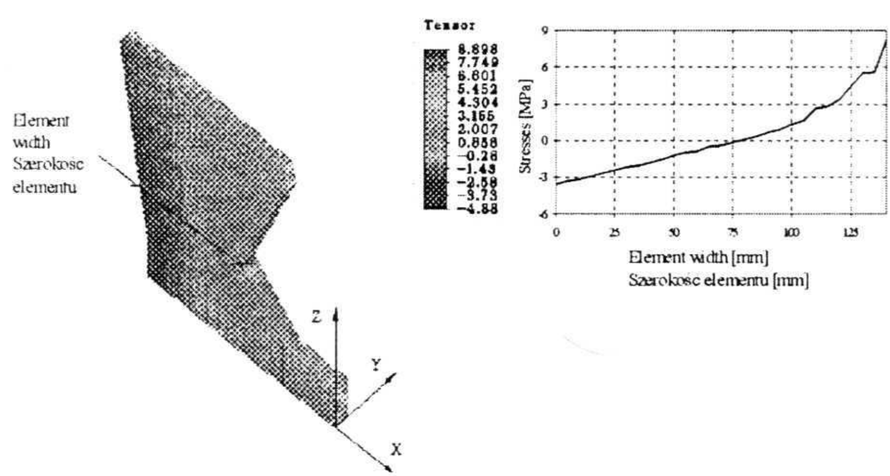


Fig. 7. Diagram of ordinary stresses in the narrowing of the side of the armchair for direction Z
Rys.7. Obraz naprężeń normalnych w przewężeniu boku fotela dla kierunku Z

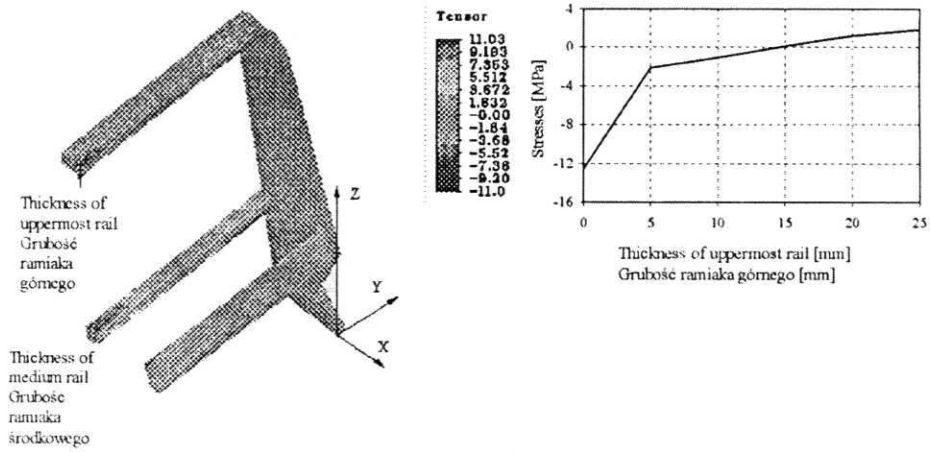


Fig. 8. Diagram of ordinary stresses in rails joining sides of the rack for direction Y
Rys.8. Obraz naprężeń normalnych w ramiakach łączących boki stelaża, dla kierunku Y

the value of wood bending strength. The pattern of ordinary stresses in individual elements is shown in Fig. 8 and 9.

Computer analysis of stresses in construction elements of the rack leads to the conclusion that the armchair requires re-designing. The side of the construction in the place where it is joined with medium rails is the spot with the highest risk of damage. Stresses caused by the application of simulated utilisation forces exceed bending strength of the board, which is hazardous during utilisation of the armchair. Optimisa-

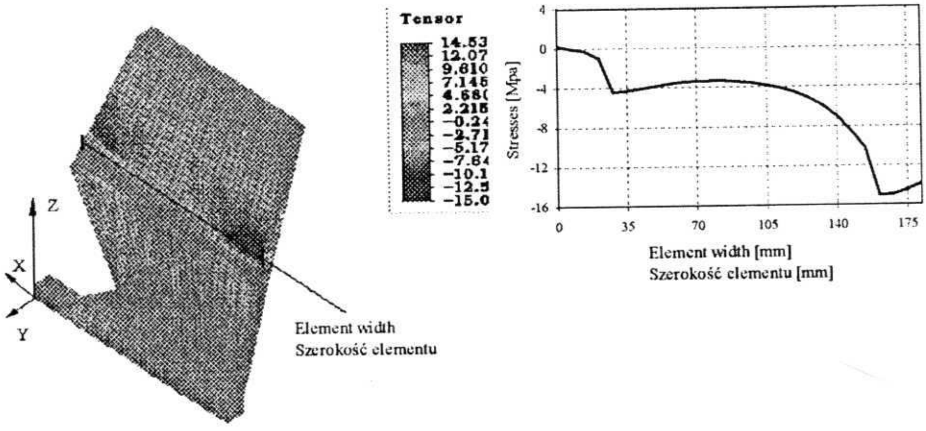


Fig. 9. Diagram of ordinary stresses in the sideboard in place where it is joined with medium rails for direction Z

Rys.9. Obraz naprężeń normalnych w płycie bocznej na wysokości łączenia z ramiakami środkowymi, dla kierunku Z

tion of the construction necessitates decreasing stresses by the application of a thicker board with improved strength parameters. The obtained results show that stresses caused in stud elements are many times lower than the bending strength of wood. This means that they are over-invested and that it is possible to use rails with lower cross section dimensions. This will allow to reduce material consumption and weight of the entire construction.

OPTIMISATION OF CONSTRUCTION ELEMENTS

Computer simulation of stresses in the construction of upholstered armchair and the analysis of the obtained results provided useful information on how to carry out the optimisation process. Optimal dimensions will be calculated for elements, in which the applied load caused the highest stresses. Other elements, in which no significant stresses were observed, will have the same dimensions as the elements for which calculations were carried out. This assumption was made because of technological reasons, namely maximal unification of dimensions of construction elements.

The following equation describing static bending strength was used to calculate optimal dimensions of construction elements:

$$\sigma = \frac{M}{W} \qquad (6)$$

$$W = \frac{bh^2}{6}$$

where:

σ – maximum bending stress equal to bending strength
M – maximum bending moment
W – section modulus
b- element breadth
h- element height.

In the above equation, σ represents bending strength of materials used for the construction. Appropriate safety modulus "m" was also considered during calculations in order to assure guaranteed utilisation reliability.

$$\sigma_{dop} = \frac{\sigma}{m} \tag{7}$$

where:

for solid wood m = 1.4
for chipboard m = 1.2

The bending moment was calculated as a product of maximum stress in a given element and section modulus,

$$M = \sigma_{max} W \tag{8}$$

OPTIMISATION OF UPPERMOST RAIL DIMENSIONS

Stress reached the highest values in this rail as a result of the application of concentrated force P_3 to the construction. Maximum stress, not exceeding the bending stress of the wood, was observed in the layers of the rail adjacent to the direct joint with the side, where $\sigma_{max} = 12.75$ MPa for b = 50 mm and h = 25 mm, provided that:

Allowable stress in elements made of solid wood amounts to:

$$\delta_{dop} = \frac{\delta}{m} = 73.4\ MPa \tag{9}$$

and maximum bending moment for the uppermost rail amounts to:

$$M = \sigma_{max} \cdot W = 66437\ Nmm \tag{10}$$

For these conditions and known proportion of h = 5/7 b, new dimensions of the rail cross section were calculated from the following equation:

$$\sigma_{dop} = \frac{6M}{bh^2} => h = 15.7\ mm \tag{11}$$

which finally allowed to calculate minimal dimensions in proportion 16x22 mm.

OPTIMISATION OF SEAT AND MEDIUM RAIL SUPPORTS

The highest stresses in the seat support resulted from the application of utilisation force 700N. Even though the maximum stress was observed at half of the support length, it was significantly lower than the bending strength of the wood, where σ_{max} = 2.25 MPa for b = 50 mm and h = 25 mm, and maximum bending moment the seat support amounted to:

$$m = \sigma_{max} \cdot W = 11719\ Nmm \tag{12}$$

For these conditions, the minimal transverse dimensions of the seat support were calculated form the following equation:

$$\sigma_{dop} = \frac{6M}{bh^2} => h = 8.8\ mm \tag{13}$$

and, considering only strength parameters, it was assumed that the optimal section should be 9x13 mm. The optimal section for medium rail was found to be 12x17 mm.

THICKNESS OPTIMISATION OF THE BOARD USED TO MAKE THE SIDE OF THE CONSTRUCTION

Maximum unit stress was observed in the joint to which a concentrated force was applied; however, the highest breaking hazard of the construction was observed in the side, at the level where it was joined with two medium rails. In this place, high stresses could be observed throughout the breadth of the side and, in joints fastening the side to the rail edges, the stress exceeded bending strength of the board and amounted to σ_{max} = 15.0 MPa for b = 180 mm and h = 16 mm, provided that:
allowable stresses in elements made of chipboard amount to:

$$\sigma_{dop} = \frac{\delta}{m} = 11.0\ MPa \tag{14}$$

and maximum bending moment for the side of the construction is:

$$m = \sigma_{max} \cdot W = 115200\ Nmm \tag{15}$$

Therefore, minimal thickness of the side of the construction for b = *constant* amounts to:

$$\sigma_{dop} = \frac{6 \cdot M}{bh^2} \Rightarrow h = 18.7\ mm \tag{16}$$

After considering strength parameters, minimal thickness of the board, which can be used to make the side of the construction, should amount to 19 mm.

OPTIMAL DIMENSIONS OF CONSTRUCTION ELEMENTS

After considering strength and technological aspects, a decision was reached that cross section dimensions of all stud elements produced of birch wood should amount to 15x25 mm. In order to obtain optimal construction, the sides of the rack should be produced of 19 mm-thick boards. It is also possible to use a thinner board with better strength parameters – for instance, a 16 mm-thick board with R_g = 15-17 MPa. Because of relatively low stresses, the front of the construction may be produced of 16 mm-thick boards. Modifications of armchair element dimensions resulted in decreased material consumption during the construction process (Table 3).

Table 3

Tabela 3

Material consumption for 1000 pieces of rack parts

Zestawienie zużycia materiału na 1000 sztuk części stelaża

Type of material Rodzaj materiału	Material consumption for 1000 pieces of product in [m^3] Zużycie materiału na 1000 sztuk wyrobów		Material saving Oszczędność materiału [%]
	Before optimisation Przed optymalizacją	After optimisation Po optymalizacji	
Solid wood Drewno	4.97	1.59	68.0
Chipboard Płyta wiórowa	9.87	11.22	-13.7

The above chart shows that optimisation results in a three-fold reduction of birch wood consumption in the process of armchair supporting structure construction. Decreased transverse dimensions of armchair elements will allow more effective utilisation of material, which was so far treated as useless wastes. The 14% increase of chipboard consumption was caused by the application of thicker materials, which was necessary in order to assure appropriate strength of the construction.

Summing up, it can be stated that the optimisation of the armchair supporting structure allowed a significant reduction of birch wood consumption and indicated that it was necessary to improve strength parameters of boards used for armchair production.

CONCLUSIONS

1. It is quite possible to reduce solid wood consumption by 68%.
2. In order to comply with safety requirements, the sides of the construction should be produced of a 19 mm-thick chipboard.

3. Application of stud elements of uniform dimensions leads to technological unification of the construction.
4. The construction fulfilled stability conditions before as well as after optimisation. Its utilisation is not hazardous for the user in this respect.
5. The performed analyses show that the application of computer techniques results in lower material consumption during the production process and assures optimal strength parameters of the armchair construction.

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OPTYMALIZACJA KONSTRUKCJI MEBLA TAPICEROWANEGO

Streszczenie

W pracy przeprowadzona została optymalizacja konstrukcji nośnej fotela jednoosobowego obejmująca stateczność oraz wymiary przekrojów poprzecznych wszystkich elementów mebla. Optymalizację przeprowadzono metodą numeryczną, przy wykorzystaniu programu komputerowego Algor. W zastosowanej metodzie uwzględnione zostały rzeczywiste właściwości mechaniczne użytych materiałów oraz

poziom wywołanych naprężeń normalnych. Uzyskane wyniki wykazały niewystarczającą wytrzymałość boku konstrukcji i duży zapas materiałowy w elementach graniakowych. Po przeprowadzeniu obliczeń uzyskano 68% oszczędność w zapotrzebowaniu na drewno lite poprzez zmniejszenie wymiarów przekroju poprzecznego wszystkich elementów z 25x50 mm na 15x25 mm. Dla zwiększenia wytrzymałości boku konstrukcji zastosowano płytę wiórową o grubości 19 mm, co nieznacznie zwiększyło zapotrzebowanie na ten materiał o niespełna 14%. Uzyskany nowy model konstrukcji nośnej zachował swoją stateczność oraz zapewnił optymalne parametry wytrzymałościowe.

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